

# HOW TO CHARACTERIZE THE NONLINEAR AMPLIFIER?

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## Abstract

The conception of the amplification of the coherent field is formulated. The definition of the coefficient of the amplification as the relation between the mean value of the field at the output to the value at the input and the definition of the noise as the difference between the number of photons in the output mode and square of the modulus of the mean value of the output amplitude are considered. On the simple example it is shown that by these definitions the noise of the nonlinear amplifier may be less than the noise of the ideal linear amplifier of the same amplification coefficient. Proposals to search the other definition of basic parameters of the monlinear amplifiers are discussed. This definition should enable us to formulate the universal fundamental lower limit of the noise which should be valid as for linear quantum amplifiers as for nonlinear ones.

## 1 Introduction

In the development of the modern communication systems the tendencies to reduce the energy of the signal and to increase their frequency take place. The question about the minimal energy of the signal which can carry the information without the significant bit error rate is important. If the amplitude coding, the phase of the field carries no information, and there is no fundamental limit on the noise of amplifiers: in principle it is possible to count the number  $n$  of photons and to construct the state of  $G^2 n$  photons, where  $G^2$  is the intensity amplification coefficient. The uncertainty of the number of photons at the output in the ideal case should be exactly  $G^2$  times greatly than at the input. But the face of the field sometimes is important. For example, if the 3-dimensional picture should be transferred, the "exact" amplification of number of photons in each mode causes the complete loss of the phase information, and the output picture is plane. So, the question of the phase properties of quantum states and their amplification is very important. For the case of the linear amplification the initially coherent state the minimal fundamental noise is determined [1] by the amplification coefficient:

$$N_{min} = GG^* - 1, \quad (1)$$

where  $N$  is the intensity of the output noise

$$N = \langle a^+ a \rangle_{out} - \langle a^+ \rangle_{out} \langle a \rangle_{out}, \quad (2)$$

while  $G$  is the amplification coefficient:

$$G = \langle a \rangle_{out} / \langle a \rangle_{in} . \quad (3)$$

Here subscripts "in" and "out" denote the initial state (before the amplification) and the output state (after the amplification). We assume that these states are related by the unitary transformation

$$| \rangle_{out} = U | \rangle_{in}; U^\dagger U = 1. \quad (4)$$

In the simplest case of the single-mode linear amplifier the transformation of the field operator is linear:

$$U^\dagger a U = G a + F, \quad (5)$$

where  $G$  is a c-number amplification coefficient and  $F$  is an operator which transforms the state of the amplifier only and does not touch the mode of the field. So, all commutators and correlators of all degrees of  $F$  and  $F^\dagger$  with all degrees of  $a$  and  $a^\dagger$  are zero.

The aim of this work is to apply the definitions (1-4) to the example of the nonlinear amplifier. (For the nonlinear amplifier the relation (5) is not valid). Here we don't interested by the squeezing properties discussed in [4],[1], the fundamental limit of the noise is the problem which should be investigated.

## 2 The parametric amplification the saturation

Consider the model example of the quantum amplifier with small nonlinearity. The parametric amplifier with the saturation of the pump may be considered as the nonlinear amplifier. In the two-mode approximation the Hamiltonian can be written in the form

$$H = i(a^\dagger b^\dagger - ab)(1 - \epsilon b^\dagger b) ;, \quad (6)$$

where  $a$  is the operator of the mode of the field,  $b$  is the operator of the idler mode, and the symbol  $:\dots:$  denotes the normal ordering. All correlators and all commutators of all degrees of  $a, a^\dagger$  with all degrees of  $b, b^\dagger$  are zero because the amplifier knows nothing about the phase of the field which should be amplified. The c-number parameter  $\epsilon$  describes the depletion of the classical pump. In the ordered form the hamiltonian (6) can be rewritten as

$$H = i(a^\dagger b^\dagger - ab - \epsilon a^\dagger b^{\dagger 2} b + \epsilon a b^\dagger b^2). \quad (7)$$

In the following consideration the parameter  $\epsilon$  is assumed to be small. Consider the transformation of the field defined by the operator

$$U = \exp(-i H t), \quad (8)$$

where  $t$  is the c-number parameter. It may be considered as dimensionless time of the interaction. The Heisenberg equations have the form

$$da/dt = b^\dagger - \epsilon b^{\dagger 2} b, \quad (9)$$

$$db/dt = a^\dagger - \epsilon(2a^\dagger b^\dagger b - ab^2). \quad (10)$$

Consider the perturbation theory by the parameter  $\epsilon$ . Let

$$a(t) = a_0(t) - \epsilon a_1(t) + O(\epsilon^2). \quad (11)$$

In the 0-th order of the perturbation theory the introduction of (11) into (9),(10) gives:

$$a_0(t) = a(0)\cosh(t) + b(0)^+ \sinh(t), \quad (12)$$

$$b_0(t) = b(0)\cosh(t) + a(0)^+ \sinh(t); \quad (13)$$

Use notations  $a = a(0)$ ,  $b = b(0)$ ,  $s = \sinh(t)$ ,  $c = \cosh(t)$ ; for the first order:

$$\begin{aligned} a_1(t) = & b^{+2}b(s^3/3 + s) + (2b^+ba + a^+b^{+2})(c^3 - 1)/3 \\ & + (a^2b + 2b^+a^+a + 2b^+)s^3/3 + (a^+a^2 + 2a)(c^3/3 - c + 2/3). \end{aligned} \quad (14)$$

Let the input we have the coherent state with amplitude  $\alpha$ ; the mean value of the number of photons  $x = \alpha\alpha^*$ . The calculation of the amplification coefficient (3) and of the mean value of photons at the output gives:

$$G = c - \epsilon(2 + x)(c^3/3 - c + 2/3) + O(\epsilon^2), \quad (15)$$

$$\langle a^+a \rangle_{out} = c^2x + s^2 - 2\epsilon(x(2 + x)c(c^3/3 - c + 2/3) + 2(x + 1)s^4/3) + O(\epsilon^2). \quad (16)$$

By the formula (1) the output noise of the amplifier is

$$N = s^2 - 4\epsilon(1 + x)s^4/3 + O(\epsilon^2), \quad (17)$$

while the noise of the ideal linear amplifier with same amplification coefficient  $G$  by formula (15) is

$$N_{lin} = G^2 - 1 = c^2 - 1 - \epsilon 2c(2 + x)(c^3/3 - c + 2/3) + O(\epsilon^2). \quad (18)$$

The difference of two last formulas gives

$$N - N_{lin} = -1.5\epsilon(c - 1)^2(2 + (c^2 + 2c + 2)x) + O(\epsilon^2). \quad (19)$$

This difference is negative at positive  $\epsilon$  and all values of  $x$ . It proves that the noise of the nonlinear amplifier may be less than the noise of the ideal linear one with the same amplification coefficient.

### 3 Discussion

The example of the "better than linear" quantum amplifier is constructed in the previous section. During the discussion at the Workshop it was suggested to consider another example of the nonlinear amplifier - the system of identical resonant 2-level atoms interacting with the single mode of the boson field. This system has the "exact" solution [3] and the dependence of the noise intensity on the amplification coefficient and the input intensity may be presented also in the extremely nonlinear case. It indicates the same possibility to realize the "better than linear" amplifier. Unfortunately, the consideration is not yet finished at the time of the deadline of the submission.

The paper on the noise properties of the "Tavis-Cummings amplifier" [3] should be published elsewhere. Formulas of the previous section based on the simplest generalization of definition of the amplification coefficient and the noise intensity to the case of the nonlinear amplifier. Formula (19) shows that in the sense of these definitions the nonlinear amplifier may be better than the ideal linear one. Either the real system with "better than linear" information capabilities can be constructed or the another, more appropriate definition of the amplification coefficient and (or) the noise of the nonlinear amplifier should be investigated. One of the alternative possibilities was suggested at the Workshop - to define the amplification coefficient by

$$G(\alpha\alpha^*) = d \langle a \rangle_{out} / d\alpha \quad (20)$$

instead of formula (3). It is under investigation now. The possibility of the other definition of the noise (instead of (2)) may be considered too. Of course, no "better than linear" amplifier should break information limits of quantum states ground in [4]. So it is important to work out appropriate parameters to characterize the nonlinear quantum amplifier of general kind.

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